INVESTIGATION OF CONTACT HEAT TRANSFER AT FLOOR SURFACES

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A solution is given for the problem of contact heat transfer at the surface of a two-layer floor in the non-stationary regime.

Contact heat transfer between a human foot and a floor may (in designing the floor) be considered as heat transfer between a finite heat source and a two-layer slab. The amount of heat absorbed by the floor from a heat source has been taken [1] as a characteristic parameter of the insulating properties of the floor.

On the basis of previous investigations [1, 2], to determine the amount of heat absorbed by a floor, we can consider the problem as one of one-dimensional heat propagation, when the temperature of the floor surface is kept constant during heat transfer by an artificial source.

The floor consists of a covering of thickness δ_1 and a base which, for short time intervals, may be assumed to be a uniform half-space relative to the covering.

It is known from [3] that the temperature field in the first layer (covering) $0 \le x \le \delta_1$ is given by

$$t_1 = t_p \left[\operatorname{erfc} \frac{x}{2\sqrt{a_1 \tau}} - h \sum_{n=1}^{\infty} h^{n-1} \left(\operatorname{erfc} \frac{2n\delta_1 - x}{2\sqrt{a_1 \tau}} - \operatorname{erfc} \frac{2n\delta_1 + x}{2\sqrt{a_1 \tau}} \right)\right]. \tag{1}$$

An expression for the specific heat flux is obtained by differentiating (1) with respect to x and substituting the result in the equality $q = \lambda_1 \left.\frac{\partial t_1}{\partial x}\right|_{x=0}$. Then

$$q = t_p \frac{b_1}{\sqrt{\pi \tau}} \left[1 + 2\sum_{n=1}^{\infty} h^n \exp\left(-\frac{n^2 \delta_1^2}{a_1 \tau}\right)\right], \tag{2}$$

where

$$h = \frac{1 - b_1/b_2}{1 + b_1/b_2}, \quad b_i = \sqrt{\lambda_i c_i \gamma_i}.$$

To determine the amount of heat Q absorbed by the floor in any time interval τ_0, we integrate (2) with respect to time τ:

$$Q = \int_0^{\tau_0} q\,d\tau = \frac{2 t_p b_1}{\sqrt{\pi}} \left[\sqrt{\tau_0} + \sum_{n=1}^{\infty} h^n \int_0^{\tau_0} \exp\left(-\frac{n^2 \delta_1^2}{a_1 \tau}\right) \frac{d\tau}{\sqrt{\tau}}\right]. \tag{3}$$

Transforming the equation by means of the substitution $x = 1/\sqrt{\tau}$ and the notation $k = n^2\delta_1^2/a_1$, and integrating the integral on the right side of (3), we obtain, returning to the original notation,

$$\int_0^{\tau_0} \exp\left(-\frac{n^2 \delta_1^2}{a_1 \tau}\right) \frac{d\tau}{\sqrt{\tau}} = 2\left[\sqrt{\tau_0} \exp\left(-\frac{n^2 \delta_1^2}{a_1 \tau_0}\right) - n\delta_1 \sqrt{\frac{\pi}{a_1}} \operatorname{erfc} \frac{n\delta_1}{\sqrt{a_1 \tau_0}}\right].$$

Substituting this value of the integral in (3), we find an expression for determining the amount of heat Q absorbed by the floor in any interval of time τ_0:

$$Q = \frac{2 t_p b_1 \sqrt{\tau_0}}{\sqrt{\pi}} \left\{1 + 2\sum_{n=1}^{\infty} h^n \left[\exp\left(-\frac{n^2 \delta_1^2}{a_1 \tau_0}\right) - n\delta_1 \sqrt{\frac{\pi}{a_1 \tau_0}} \operatorname{erfc} \frac{n\delta_1}{\sqrt{a_1 \tau_0}}\right]\right\}. \tag{4}$$

If the thermal coefficients of the first and second layers are identical, then h = 0, and we obtain the same solution as for a single half-space [4]:

$$Q = 2t_p b_1 \sqrt{\tau_0}/\sqrt{\pi}\,.$$

Equation (4) can be used to determine the heat losses from a source maintaining a constant temperature at the surface of the floor.

NOTATION

a_1 – thermal diffusivity of first layer; t_1 and t_2 – temperature of covering and base; t_p – floor temperature at initial moment of time, kept constant during the heat transfer process; b_1 and b_2 – heat accumulation coefficients of first and second layers; Q – amount of heat in time τ_0.

REFERENCES

1. I. S. Cammerer and W. Schüle, Gesundheits-Ingenieur, no. 1, 1960.
2. W. Schüle, Gesundheits-Ingenieur, no. 23-24, 1954.
3. A. V. Lykov, Theory of Heat Conduction [in Russian], Gostekhizdat, 1952.
4. G. Gröber, S. Erk, and U. Griguli, Fundamentals of Heat Transfer [Russian translation], IL, 1958.

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